INTERFERENCE IN THIN FILMS

When a film of oil spreads over the surface of water, or a thin glass plate is illuminated by light, interference occurs between the light waves reflected from the film, and also between the light waves transmitted through the film.

Let a monochromatic light ray SP_1 be incident at an angle **i** on a parallel-sides transparent thin film of thickness t and refractive index μ**.** At the point P_1, the ray is partly reflected along P_1R_1 and partly refracted along P_1C_1 at angle r**.** At point C_1

It is again partly reflected along C_1P_2 and partly refracted along C_1T_1. Similarly reflection and refractions occur at pointsP_2C_2.....etc as shown (Fig-a)**.**Thus we get a set of parallel reflected rays P_1R_1 P_2R_2 ....etc. and a set of parallel transmitted rays C_1T_1, C_2T_2...etc. At any boundary between two different transparent media there is always partial reflection and partial transmission**.**

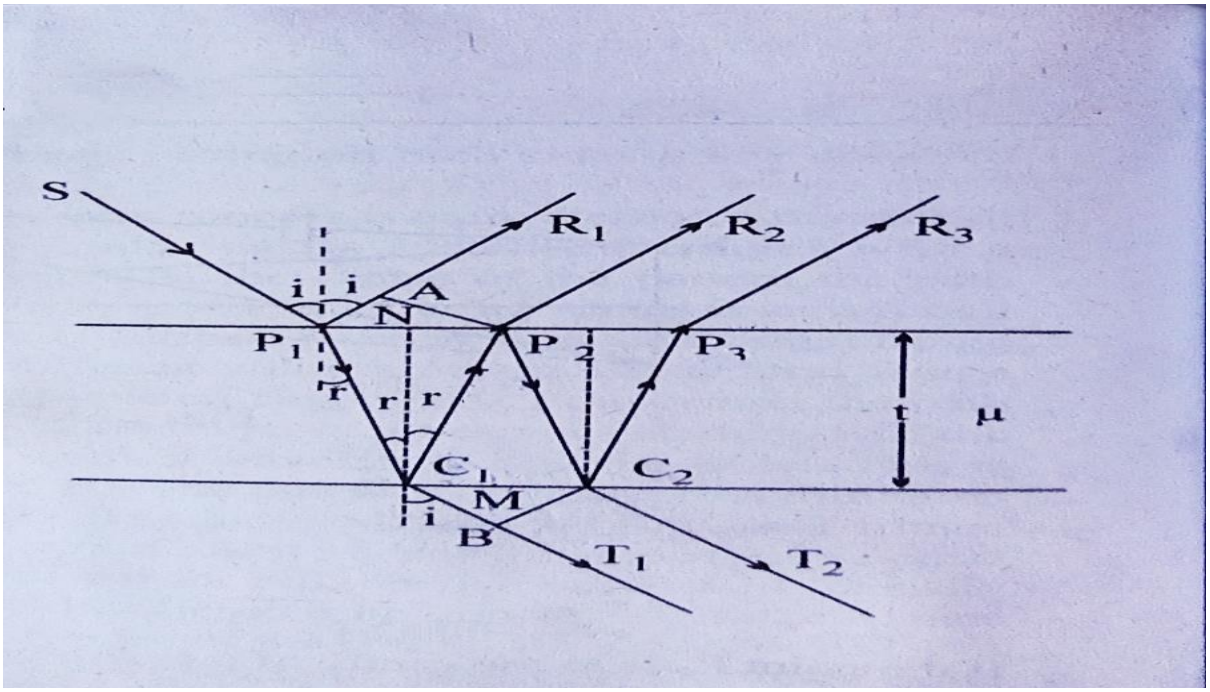

FIG. - a

Interference in reflected rays

The reflected rays are P_1R_1 and P_2R_2, let P_2A is the perpendicular draw from P_2 to P_1R_1**.** Since path difference in reflected rays from P_1R_1 to P_2R_2

Path difference = path $P_1C_1P_2$ in medium – path P_1A in air

$$\Delta_1 = \mu (P_1C_1 + C_1P_2) - P_1A \qquad \text{………. (1)}$$

Angle of refraction r = $\angle P_1C_1N = \angle NC_1P_2$

$$P_1C_1 = C_1P_2 = \frac{t}{\cos r} \qquad \text{………. (2)}$$

$$i + \angle AP_1P_2 = \angle AP_1P_2 + \angle P_1P_2A = 90°$$

$$i = \angle P_1P_2A$$

Hence $P_1A = P_1P_2 \sin i$

$$= (P_1N + NP_2)\sin i$$

But $P_1N = NP_2 = t \tan r$

$$P_1A = 2 \tan r \sin i \qquad \text{…………… (3)}$$

Substituting the value of equation (3) and (2) in en.(1), we get

$$\Delta_1 = 2\mu \frac{t}{\cos r} - 2t \tan r \sin i$$

$$= 2\mu \frac{t}{\cos r}\left[1 - \frac{\tan r \sin i \cos r}{\mu}\right]$$

But $\mu = \frac{\sin i}{\sin r}$

$$\Delta_1 = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta_1 = 2\mu \cos r \qquad \text{…………(4)}$$

Now according to stokes law, if the ray P_1R_1 suffer a reflection at the surface of a denser medium under goes a plane change of π which is equivalent to a path difference of $\lambda/2$. At point C_1 the reflection takes place when the ray P_1P_2 is going from a denser to rarer medium. Hence the effective path difference between P_1R_1 and P_2R_2 is

$$\Delta = \Delta_1 - \lambda/2 = 2\mu t \cos r - \lambda/2$$

(i) **Constructive interference**: We know that for constructive interference the path difference should be $\Delta = n\lambda$ (where n = 0,1,2,3…….)

When $2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

Or $\mathbf{2\mu t \cos r = (n+\frac{1}{2})\lambda}$ ………(5)

When this condition is satisfied the film will appear bright in the reflected light .

(ii) **Destructive interference** : For destructive interference the path difference should be $\Delta = (2n \pm 1)\frac{\lambda}{2}$

When $2\mu t \cos r - \lambda/2 = (2n - 1)\frac{\lambda}{2}$

Or $\mathbf{2\mu t \cos r = n\lambda}$ ………..(6)

When this condition is satisfied the film will appear dark in the reflected light.

(b) Interference in transmitted rays

The transmitted rays are C_1T_1 and C_2T_2. Let C_2B is the perpendicular draw from C_2 to C_1T_1.

Since path difference in transmitted ray from T_1 and T_2

$$\Delta = \text{path } C_1P_2C_2 \text{ in medium} - \text{path } C_1B \text{ in air}$$

$$= \mu(C_1P_2 + P_2C_2) - C_1B \quad \text{…………..(7)}$$

Angle of refraction $r = \angle C_1P_2M = \angle C_2P_2M$

$$C_1P_2 = C_2P_2 = \frac{t}{\cos r} \quad \text{……………(8)}$$

$$i + \angle BC_1C_2 = \angle BC_1C_2 + \angle BC_2C_1 = 90°$$

$$\angle i = \angle BC_2C_1$$

$$C_1B = C_1C_2 \sin i$$

$$= (C_1M + MC_2)\sin i$$

But $C_1M = C_2M = t \tan r$

$$C_1B = 2t \tan r \sin i \quad \text{……………….(9)}$$

Substituting the value of equation (8) and (9) in equation (7),we get

$$\Delta = \frac{2\mu t}{cosr} - 2t \tan r \sin i$$

$$= \frac{2\mu t}{cosr}\left[1 - \frac{tanr\ sini\ cosr}{\mu}\right]$$

But $\mu = \frac{sini}{sinr}$

$$\Delta = \frac{2\mu t}{cosr}(1 - \sin^2 r)$$

$$\Delta = 2\mu t \cos r \qquad \text{…………(10)}$$

In this case, there is no phase change due to reflection at C_1P_2 because the light is travelling from denser to rarer medium. Hence the effective path difference between C_1T_1 and C_2T_2 is

$$\mathbf{\Delta = 2\mu t \cos r}$$

(i) **Constructive interference**: For constructive path difference

$$\Delta = n\lambda$$

$$\mathbf{2\mu t \cos r = n\lambda} \qquad \text{……………….(11)}$$

The film will appear bright in the transmitted light.

(ii) **Destructive interference**: For destructive interference

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\mathbf{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}} \qquad \text{……………….(12)}$$

Where n = 0,1,2,3………

And the film will appear dark in the transmitted light.

(c) Complementary interference

A comparison of (5), (6), (11) and (12) shows that the conditions for constructive and destructive interference in the reflected light are just the reverse of those in the transmitted light. Hence the film which appears bright in reflected light will appear dark in transmitted light and vice-versa. In other words, the appearances in the two cases are complimentary to each other.

(d) Interference due to an infinitely thin film

When the thin of the film is small compared to the wave length of light (λ >> t). The effective path difference between the interfering waves in reflected light for a film is 2ut cosr - λ/2. When the film is excessively thin such that its thickness t is very small compared to the wavelength of light then 2ut cos r is almost zero. Hence the effective path difference becomes λ/2. This is condition for minimum intensity. Hence every wave length will be absent and the film will appear black in reflected light and it will appear bright in transmitted light.

(e) Interference in a thick film

When the thickness of the film is large compared to the wave length of light (t>> λ), the path difference at any point of the film will be large. In this case the condition of constructive interference (**$2\mu t \cos r = (n+\frac{1}{2})\lambda$**),at a given point is satisfied by large number of wave length with the value of n different for different colours. At the same point the condition of destructive interference (**$2\mu t \cos r = n\lambda$**) is also satisfied for another set of large number of wavelength .The

Thus in the case of a thick film illuminated by white light,the colors are not observed in the reflected light and its appears uniformly illuminated.

(f) Colors of thin films:

When a thin film is illuminated by monochromatic light and seen in reflected light, it will appear bright if **$2\mu t \cos r = (n+\frac{1}{2})\lambda$** *and dark if* **$2\mu t \cos r = n\lambda$.** *however the film is illuminated by white light the film shows different colors.*

The eye looking the film receives the waves of light reflected from the upper and lower surfaces of the film. For a thin film these rays are very close to each other. The path difference between interfering rays is (**$2\mu t \cos r - \frac{1}{2}\lambda$**). The path difference depends upon t, the thickness of the film and upon r which depends upon the inclination of the incident rays. Now white light consist of a continuous range of wave lengths. For a particular value of t and r i.e. at a particular point of the film and for a particular position of the eye, the waves of only certain wavelength satisfy the constructive interference. Therefore only those color will be present in the reflected system with maximum intensity. The other neighbouring wavelengths will be present with less intensity. There will also be certain wavelengths which satisfy the condition of minima. Such wavelengths of colors will be absent from the reflected light. AS result the point of the film will appear colored.

(g) Fringer of equal inclination or Haidinger fringes

Let us consider that a thin film is illuminated by an extended monochrometic light source. When the film is of uniform thickness, the path difference **2μt cosr** between the coherent beams is only due to the change in r. If the thickness of a film is large, the path ifference will change appreciably even when r changes in a very small way. In this case each fringe represent the locus of all point on the film, ray from which are equally inclined to the normal. There fringes are called fringes of equal inclination. The fringes of equal inclination are known as Haidinger fringes. In this case all the pairs of interfering rays of equal inclination pass through the plate as a parallel beam and hence meet at infinity. The other pairs of different inclination meet at different points at infinity.

Necessity of an Extended Light Source

An extended source is necessary to enable the eye to see whole of the film and observed the interference pattern due to whole film.

Let us consider a thin film and a narrow source of light at S (fig.a) then for each incident ray E_1,E_2 we get a pair of parallel interfering rays. The incident ray produces interference fringes because E_{1r},E_{1r}' reach the eye where as the incident ray E_2 meet the surface at some different angle and is reflected along E_{2r} E_{2r}'. Here E_{2r} E_{2r}' do not reach the eye. Therefor,the position A of the film is visible.

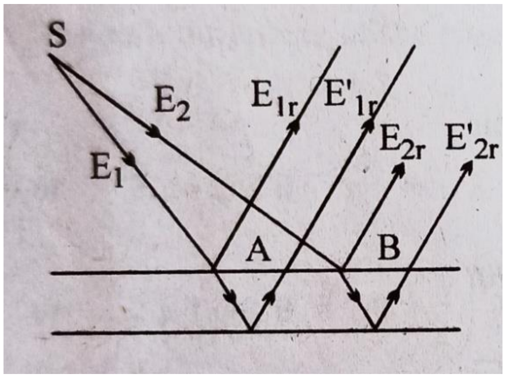

Fig – a

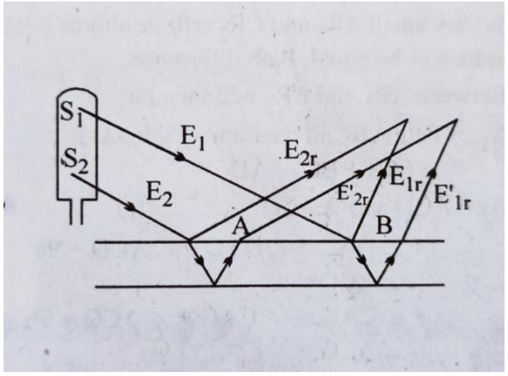

Fig - b

If extended source of light is used (fig.b)the incident ray E_1 after reflection from the upper and the lower surface of the film emerges as E_{1r},$E_{1r'}$ which reach the eye. Also incident ray E_2 from some other point of the source after reflection from the upper and lower surface of the film emerges as E_{2r},E_{2r}' which also reach the eye. Therefore,in the case of such a source of light, the rays incident at different angles on the film are well adjust by the eye and the field of view is large. Due to this reason, to observe interference phenomenon in this film, a broad source of light is required. With a broad *source* of light, rays of light are incident at different angels and the reflected parallel beams reach the eyes or the microscope objective. Each such ray a light has its origin at a different point on the source.

Interference due to a thin wedge – shaped film

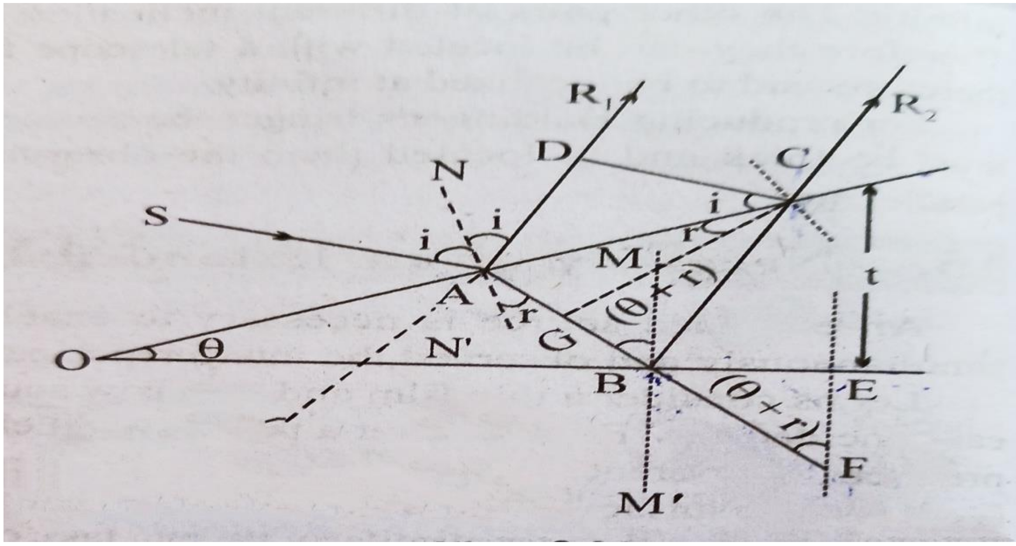

Fig – a

Let us consider a thin wedge shaped film of refractive index μ, bounded by two plane surfaces inclined at an angle (fig. a). Let the film be illuminated by a parallel beam of monochromatic light the interference occurs between the incident ray reflected at the upper and lower surfaces of the film so that equidistant alternate dark and bright fringes becomes visible. The interfering rays in reflected light are AR_1 and CR_2, both originating from the same incident ray SA. To evaluate the path difference between these two rays the perpendiculars CD and CG are drawn from C on AR_1 and AB. If θ is an angle of wedge – shaped film is very small AR_1 and CR_2 will be almost parallel and after the perpendicular CD the paths will be equal.

The path difference between AR_1 and CR_2 will be

Δ_1= path ABC in medium – path AD in air ………..(1)

= μ(AB+BC) – AD

As ΔACD and ΔACG

$i+\angle DAC = \angle DAC + \angle ACD = 90°$

$\angle ACD = i$

$r + \angle CAG = \angle CAG + \angle ACG = 90°$

$\angle ACG = r$

Refractive index μ of the film $\mu = \frac{\sin i}{\sin r} = \frac{AD/AC}{AG/AC} = \frac{AD}{AG}$

AD = μ AG ………….. (2)

To find the length of the path of light ray BC , now draw perpendicular CE from C on OE and produces AB. These meets at F.

We know that exterior angle = sum of the interior angle

Hence $\angle ABE = \angle MBA + 90°$

$= \theta + 90 + r$

$\angle MBA = (\theta + r)$

$(\theta + r) = \angle MBA = \angle M'BF = \angle BFC$

$= \angle MBC = \angle BCF$

Hence ΔBCF is the isosceles triangles.

BC = BF

CE = EF =t ……………..(3)

Substituting the value of equation (2) and (3) in equation (1), we get

$\Delta_1 = \mu$ (AB+BF – AG)

$= \mu\, FG$

$= \mu CF \cos(\theta + r)$

$\Delta_1 = 2\, \mu t \cos(\theta + r)$

The ray is reflected from a denser medium a phase change of π or a path difference is $\lambda/2$. Hence between the interfering rays AR_1 and CR_2 is

$\Delta = \Delta_1 - \lambda/2$

$\Delta = 2\mu t \cos(\theta + r) - \lambda/2$

If incident ray of light is perpendicular to the wedge-shaped film

$\angle i = \angle r = 0$

Then the effective path difference $\Delta = 2\mu t \cos\theta - \lambda/2$

Constructive interference

The path difference for constructive interference is

$\Delta = n\lambda$ (n = 0,1,2,3,…….)

Or $2\mu t \cos\theta - \frac{\lambda}{2} = n\lambda$

$$2\mu t \cos\theta = (2n+1)\, \lambda/2$$

When this condition is satisfied,the film will appear bright in the refracted light.

Destructive interference

The path difference for the destructive interference is

$$\Delta = (n \pm \frac{1}{2})\lambda$$

$$2\mu t \cos\theta - \frac{\lambda}{2} = (n - \frac{1}{2})\lambda \qquad (n=1,2,\ldots\ldots)$$

$$2\mu t \cos\theta = n\lambda$$

Under this condition the film will appear dark in the reflected light.

While r and θ are very small. So the path difference primarily depends on the thickness of the film t.

The thickness of film in wedge-shaped film increases from zero as the distance from the edge increases. Therefore the path difference between the interfering rays increases from zero with the increasing of the thickness of film.Thus condition (7) and (8) are satisfied .Alternately as the thickness of the film increases. Hence dark and bright fringes are alternatively in the wedge shaped film.

(i) Fringe width

Let (n+1) and n dark fringes be formed at distance x_{n+1} and x_n from the wedge 0.

Fringe width $\beta = (x_{n+1} - x_n)$

If the thickness of film at these distance are t_{n+1} and t_n, then

$$t_{n+1} = x_{n+1} \tan\theta$$

$$t_n = x_n \tan\theta$$

or $$t_{n+1} - t_n = (x_{n+1} - x_n) \tan\theta = \beta \tan\theta$$

For normal or near normal incidence $i \approx 0, r \approx 0$

For n^{th} dark fringe

$$2\mu t_n \cos\theta = n\lambda$$

For $(n+1)^{th}$ dark fringe

$$2\mu t_{n+1} \cos\theta = (n+1)\lambda$$

Subtracting equation (10) from equation (11)

$$2\mu(t_{n+1} - t_n)\cos\theta = \lambda$$

Substituting the value in equation (9)

$$2\mu\beta\tan\theta\ \cos\theta = \lambda$$

$$2\mu\beta sin\theta = \lambda$$

So that $\beta = \frac{\lambda}{2\mu sin\theta} \approx \frac{\lambda}{2\mu\theta}$ (if wedge angle is very small $sin\theta \approx \theta$)

$$\beta = \frac{\lambda}{2\mu\theta}$$

From this equation it is clear that the wedge film is form an angle θ is very small then the fringe width of the reflected light is independent of n. Therefore for dark fringe and bright fringes are equal width and if the wedge angle θ is increase then fringe width is also decrease and if wavelength is increase then fringe width is also increase.

Fringes of equal thickness or fizeau fringes

The path difference $\Delta = 2\mu t\ cos\theta$ between the rays reflected from the wedge shaped film of constant wedge angle depends on the thickness of the film at that place where light is incident normally on the film. Due to this reason the interference fringes will be the locus of all those points at which the thickness t of film has a constant value. So there fringes are straight fringes of equal thickness called **Fizeau Fringes.**

The fringes width of these fringes $\beta = \frac{\lambda}{2\mu\theta} is$ depend on the wedge angle while it is not depend on the thickness of film. These fringes are formed within film. So they are also called localized fringes. This types of fringes are observed in Michelson's interferometer

Testing the plane of surfaces

For testing the planeness of surface of a glass plate it is placed on an optically plane surface in such a way that a wedge shaped air film of very small wedge angle is formed between these surfaces. Now it is illuminated with monochromatic light. The fringes so produces, are observed in the study of microscope and if the fringes are of equal width and straight, it means that the testing surface is plane otherwise it is not plane.