[This question paper contains 8 printed pages.]

Your Roll No. ....

Sr. No. of Question Paper : 3071

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (Hons) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

P.T.O.

3071

1. (a) Define Convexity of a bond and find the relation between the Convexity, Duration and Bond price. How does convexity measure sensitivity of the portfolio?

(b) Consider the three bonds having payments as shown in the table below. They are traded to procure a 12% yield with continuous compounding.

End of year payments	Bond A	Bond B	Bond C
Year 1	1000	500	0
Year 2	1000	500	0
Year 3	1000+10000	500+10000	0+10000

Determine the price and duration of each bond.

(c) Explain Forward Rates. Derive the following relation :

$$R_F = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where R_F is the forward interest rate for the period between T_1 and T_2. R_1 and R_2 are the zero rates for maturities T_1 and T_2 respectively. What happens when $R_2 > R_1$?

2. (a) (i) An investor sells a European call option with strike price of K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

(ii) An investor sells a European call on a share for ₹6, the stock price is ₹45 and the strike price is ₹52. Under what circumstances will the seller of the option make a profit? Under what circumstances will the option be exercised?

(b) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(c) Explain Hedging. A United States company expects to pay 1 million Euros in 3 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the gamma of a European call and the gamma of a European put on a non-dividend-paying stock.

(c) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments, ($e^{-0.005} = 0.9950$).

4. (a) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ₹38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and A shares of the stock. What is the value of A which makes the

portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0.005} = 1.005$).

(b) A stock price is currently ₹50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹51? (You can use exponential value: $e^{-0.0125} = 0.9876$).

(c) Consider a two-period binomial model with current stock price S_0 = ₹100, the up factor $u = 1.2$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹104 and maturity $T = 1$ year. ($e^{-0.025} = 0.9753$)

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right],$$

where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(S_T > K)$, the probability that a European call option be exercised in a risk-neutral world.

(b) Show that the Black—Scholes-Merton formulas for call and put options satisfy the put-call parity.

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹52, the strike price is ₹50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is three months?

(You can use values: $\ln(26/25) = 0.0392$, $\exp(-0.03) = 0.9704$).

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

(b) Companies A and B have been offered the following rates per annum on a ₹10 million loan for 5 years :

	Fixed rate	Floating rate
Company A	12.0%	LIBOR+ 0.1%
Company B	14.5%	LIBOR + 0.9%

Company A requires a floating-rate loan; Company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

(c) Find the payoff from a bull spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 4 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 3182

Unique Paper Code : 32357610

Name of the Paper : DSE-4(i): Number Theory

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) A farmer purchased 100 head of livestock for a total cost of Rs. 4000. Prices were as follow: calves, Rs.120 each; lambs, Rs.50 each; piglets, Rs.25 each. If the farmer obtained at least one animal of each type, how many of each did he buy? (6.5)

(b) Define a complete set of residues modulo n. Verify that 0, 1, 2, 2^2, 2^3, ⋯, 2^9 form a complete set of residues modulo 11, but that 0, 1^2, 2^2, 3^2, ⋯ 10^2 do not. (6.5)

(c) Obtain three consecutive integers, the first of which is divisible by a square, the second by a cube, and the third by a fourth power. (6.5)

2. (a) State and prove Wilson's theorem. What about its converse? Justify your answer. (6.5)

(b) (i) Use Fermat's theorem to show that if p is an odd prime, then

$1^p + 2^p + 3^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$.

(ii) If p and q are distinct primes, prove that $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$. (6.5)

(c) Define Mobius μ-function. If the integer n > 1 has the prime factorization $n = p_1^{k_1} p_2^{k_2} \cdots p_2^{k_r}$ then prove that

$$\sum_{d|n} \frac{\mu(d)}{d} = \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_r}\right).$$ (6.5)

3. (a) Find the highest power of 5 and the highest power of 2 in 1000! and hence find the number of zeros with which the decimal representation of 1000! terminates. (6.5)

(b) Prove that for $n > 1$, the sum of the positive integers less than n and relatively prime to n is $\frac{1}{2}n\phi(n)$ and hence find the sum of the positive integers less than 100 and relatively prime to 100. (6.5)

(c) Define Euler's ϕ-function. Show that for each positive integer $n \geq 1$,

$$n = \sum_{d|n} \phi(d)$$

the sum being extended over all positive divisors of n. Verify this result for $n=32$. (6.5)

4. (a) Show that if $\gcd(m,n) = 1$, where $m > 2$ and $n > 2$, then the integer mn has no primitive roots. Hence deduce that 21 has no primitive roots. (6.5)

(b) Define primitive root of the integer $n > 1$. Find all the primitive roots of 25. (6.5)

(c) State Euler's criterion to determine whether an integer a is a quadratic residue of a given prime p. Also show that 3 is a quadratic residue of 13 but a quadratic nonresidue of 17. (6.5)

5. (a) Show that if r is a primitive root of the prime $p \equiv 1 \pmod 4$, then $r^{(p-1)/4}$ satisfies the quadratic congruence $x^2 + 1 \equiv 0 \pmod p$. (6.5)

(b) Obtain the solution of the quadratic congruence

$$x^2 \equiv 23 \pmod{7^2}$$ (6.5)

(c) Let p be an odd prime and let a and b be integers that are relatively prime to p. Then prove that $(ab/p) = (a/p)(b/p)$.

Further determine whether the congruence $x^2 \equiv -46 \pmod{17}$ is solvable. (6.5)

6. (a) When the RSA algorithm is based on the key $(n,k) = (1537,47)$, what is the recovery exponent for the cryptosystem? (5)

(b) Decrypt the message HOZTKGH, which was produced using the linear cipher $C \equiv 3P + 7 \pmod{26}$. (5)

(c) Use the Hill's cipher

$$C_1 \equiv 5P_1 + 2P_2 \pmod{26}$$
$$C_2 \equiv 3P_1 + 4P_2 \pmod{26}$$

to encipher the message GIVE THEM TIME. (5)

[This question paper contains 8 printed pages.]

June 2024

Your Roll No..............

H

Sr. No. of Question Paper : 3183

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each questions.
3. **All** questions carry equal marks.

1. (a) Define a Convex Set. Show that the set S defined as :

$$S = \{(x, y) \mid y^2 \leq 4x\} \text{ is a Convex Set.}$$

(b) Let $x_1 = 1$, $x_2 = 2$, $x_3 = 4$ be a feasible solution to the system of equations :

$$2x_1 + 3x_2 - x_3 = 4$$

$$3x_1 - x_2 + x_3 = 5$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Consider the following linear programming problem:

Minimize $z = cx$

subject to $Ax = b$, $x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution with objective function value z_B corresponding to a basis B where $x_B = B^{-1}b$. By entering an a_j with $z_j - c_j > 0$ and removing a b_r subject to :

$$\frac{x_{Br}}{y_{rj}} = \text{Min}\left[\frac{x_{Bi}}{y_{ij}} : y_{ij} > 0\right]$$

Show that we can get a new feasible solution with improved value of the objective function compared to z_B.

2. (a) Using Simplex method, find the solution of the following linear programming problem :

Maximize $z = x_1 - 2x_2 + x_3$

subject to $x_1 + 2x_2 + x_3 \leq 12$

$2x_1 + x_2 - x_3 \leq 6$

$x_1 - 3x_2 \geq -9$

$x_1, x_2, x_3 \geq 0.$

(b) Using two phase method, solve the linear programming problem :

Minimize $z = -3x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0.$

(c) Solve the following linear programming problem by Big - M method :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0.$

3. (a) Consider the following primal problem (P) and dual problem (D) :

(P) Minimize $z = cx$

Subject to $Ax \geq b$, $x \geq 0$

(D) Maximize $z = wb$

Subject to $wA \leq c$, $w \geq 0$,

If x_0 (w_0) is an optimal solution to the primal (dual) problem then there exists a feasible solution $w_0(x_0)$ to the dual (primal) such that $cx_0 = w_0b$.

(b) Use graphical method to solve the dual of the following linear programming problem :

Minimize $z = 6x_1 + 8x_2 + 7x_3 + 15x_4$

Subject to $x_1 + x_3 + 3x_4 \geq 4$

$x_2 + x_3 + x_4 \geq 3$

$x_1, x_2, x_3, x_4 \geq 0$

Further, find an optimal solution to the given problem from optimal solution of the dual problem.

(c) Obtain the dual of the following linear programming problem :

Maximize $z = 10x_1 + x_2 + 2x_3$

Subject to $x_1 + x_2 - 2x_3 \geq 10$

$x_1 + 4x_2 - 3x_3 = 3$

$4x_1 + x_2 + x_3 \leq 20$

$x_1 \geq$, $x_2 \leq 0$, and x_3 unrestricted in sign.

4. (a) A Company has four warehouses, a, b, c and d. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following amounts in stock.

Warehouse :	a	b	c	d
No. of units :	15	16	12	13

and the customer's requirements are

Customers :	A	B	C
No. of units	18	20	18

The table below shows the costs of transporting one unit from warehouses to the customers :

	a	b	c	d
A	8	9	6	3
B	6	11	5	10
C	3	8	7	9

Find the optimal schedule and minimum total transport cost.

(b) A Company is faced with the problem of assigning six different machines to six different jobs. Determine the optimal solution of the Assignment Problem with the following cost matrix :

	a	b	c	d	e	f
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	3	56	53	31	17	28

(c) For the following cost minimization Transportation Problem, find initial basic feasible solution by using North-West comer rule, Least Cost method and Vogel's approximation method. Compare the three solutions (in terms of cost).

	A	B	C	D	Supply
I	19	14	23	11	11
II	15	16	12	21	13
III	30	25	16	39	19
Demand	6	10	12	15	

5. (a) Define the Saddle point. The pay-off matrix of a game is given below. Find the best strategy for each player, and the value of a play of the game of A and B.

Player B

Player A

	I	II	III	IV	V
I	9	3	1	8	0
II	6	5	4	6	7
III	2	4	3	3	8
IV	5	6	2	2	1

(b) Convert the following Game problem into a linear programming problem for Player A and Player B and solve it by Simplex method.

Player B

Player A	3	-2	4
	-1	4	2

(c) Using Simplex method, solve the system of equations :

$$3x_1 + x_2 = 7$$

$$x_1 + x_2 = 3$$

Also write the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.