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Sr. No. of Question Paper : 2915

Unique Paper Code : 32351201

Name of the Paper : Real Analysis (CBCS-LOCF)

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Let S be a nonempty bounded set in $\mathbb{R}$. Let $b < 0$, and let $bS = \{bs: s \in S]$. Prove that $\inf(bS) = b(\sup S)$ and $\sup(bS) = b(\inf S)$. (6.5)

   (b) State and prove Archimedean Property of real numbers. (6.5)

(c) Prove that $(0,1]$ is neither an open set nor a closed set. (6.5)

2. (a) If $a, b \in \mathbb{R}$, then prove that $||a| - |b|| \leq |a - b| \leq |a| + |b|$. (6)

(b) Prove that an upper bound u of a nonempty set S in $\mathbb{R}$ is the supremum of S if and only if for every $\varepsilon > 0$ there exists an $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$. (6)

(c) If S is nonempty subset of $\mathbb{R}$, show that S is bounded if and only if there exists a closed and bounded interval I such that $S \subseteq I$. (6)

3. (a) Prove that $\lim_{n\to\infty} x_n = 0$ if and only if $\lim_{n\to\infty} |x_n| = 0$. Give an example to show that the convergence of $\langle |x_n| \rangle$ need not imply convergence of $\langle x_n \rangle$. (6.5)

(b) Prove that every monotonically increasing bounded above sequence is convergent. (6.5)

(c) Define Cauchy sequence. Show directly using the definition that

$\left\langle 1 + \frac{1}{2!} + \cdots\cdots + \frac{1}{n!} \right\rangle$ is a Cauchy sequence. (6.5)

4. (a) If $\langle a_n \rangle$ and $\langle b_n \rangle$ converges to a and b respectively, prove that $\langle a_n + b_n \rangle$ converges to $(a+b)$. (6)

(b) Show that a sequence in $\mathbb{R}$ can have at the most one limit. (6)

(c) Show that $\lim\limits_{n\to\infty} c^{1/n} = 1$ for $c > 0$. (6)

5. (a) Examine the convergence or divergence of the following series.

(i) $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\cdots$

(ii) $\frac{\sqrt{2}}{3.5}+\frac{\sqrt{4}}{5.7}+\frac{\sqrt{6}}{7.9}+\frac{\sqrt{8}}{9.11}+\frac{\sqrt{10}}{11.13}+\cdots$ (6.5)

(b) Prove that $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ is a telescoping series, and find its sum. (6.5)

(c) Prove that the p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, $p>0$ converges for $p>1$ and diverges for $p\le 1$. (6.5)

6. (a) State and prove ratio test (limit form). (6)

(b) Examine the convergence of the following series, clearly stating the test used

$$\frac{1}{e}+\frac{4}{e^2}+\frac{27}{e^3}+\frac{256}{e^4}+\frac{3125}{e^5}+\cdots$$ (6)

(c) Define Alternating series of real numbers. Test for the absolute convergence and convergence of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ (6)

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H

Sr. No. of Question Paper : 4121

Unique Paper Code : 2352011202

Name of the Paper : CALCULUS – DSC 5

Name of the Course : **B.Sc. (H) Mathematics UGCF-2022**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **three** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) If f: A → $\mathbb{R}$ and if c is a cluster point of A then prove that f can have only one limit at c. (5)

   (b) Use $\in - \delta$ definition of limit to establish the following limit : (5)

$$\lim_{x \to -1} \frac{x+5}{2x+3} = 4.$$

P.T.O.

(c) Determine whether the following limit exists in $\mathbb{R}$

$$\lim_{x \to 0} sgn \sin\left(\frac{1}{x}\right)$$ (5)

(d) Let $A \subseteq \mathbb{R}$, let $f, g, h: A \to \mathbb{R}$, and let $c \in \mathbb{R}$ be cluster point of A. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x \to c} f = L = \lim_{x \to c} h$, then show that $\lim_{x \to c} g = L$. (5)

2. (a) State and prove sequential criterion for continuity. (5)

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is discontinuous at every x in $\mathbb{R}$. (5)

(c) Let f and g be continuous real-valued function on (a, b) such that $f(r) = g(r)$ for each rational number r in (a, b) then prove that $f(x) = g(x)$ for all $x \in (a, b)$. (5)

(d) State Intermediate Value Theorem. Show that $x = \cos x$ for some x in $\left(0, \frac{\pi}{2}\right)$. (5)

3. (a) Prove that every continuous function defined on a closed interval is bounded therein. (5)

(b) If f is uniformly continuous on a set S and (s_n) is a Cauchy sequence in S, then prove that $(f(s_n))$ is a Cauchy sequence. (5)

(c) Show that the function $f(x) = \frac{1}{x^2}$ is not uniformly continuous on (0,1) but it is uniformly continuous on $[a, \infty)$ where $a > 0$. (5)

(d) Let $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show that f is differentiable on $\mathbb{R}$ and also show that f′ is not continuous at $x \neq 0$. (5)

4. (a) Let f be defined on an open interval containing x_0. If f assumes its maximum or minimum at x_0 and if f is differentiable at x_0 then show that $f'(x_0) = 0$. (5)

(b) State Intermediate Value Theorem for derivatives. Suppose f is differentiable on E and $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that $f'(x) = \frac{1}{3}$ for some $x \in (0,2)$.

(ii) Show that $f'(x) = \frac{2}{5}$ for some $x \in (0,2)$.

(5)

(c) Show that $ex \leq e^x$ for all $x \in \mathbb{R}$. (5)

(d) Suppose f is differentiable on $\mathbb{R}$, $1 \leq f'(x) \leq 2$ for $x \in \mathbb{R}$, and $f(0) = 0$. Prove $x \leq f(x) \leq 2x$ for all $x \geq 0$. (5)

5. (a) Let f be differentiable function on an open interval (a, b). Then show that f is strictly increasing on (a, b) if $f'(x) > 0$. (5)

(b) If $y = \sin^{-1} x$, prove that (5)

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - n^2 y_n = 0.$$

(c) If $y = \left[x + \sqrt{1+x^2}\right]^m$, find $y_n(0)$. (5)

(d) State Taylor's theorem. Find Taylor series expansion of cos x. (5)

6. (a) Find all values of k and l such that

$$\lim_{x \to 0} \frac{k + \cos lx}{x^2} = -4.$$ (5)

(b) Determine the position and nature of the double points on the curve (5)

$$y^2 = (x - 2)^2(x - 1).$$

(c) Sketch a graph of the rational function showing the horizontal, vertical and oblique asymptote (if any) of $y = x^2 - \frac{1}{x}$. (5)

(d) Sketch the curve in polar coordinates of $r = \cos 2\theta$. (5)

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Sr. No. of Question Paper : 4159 H

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : B.Sc. (Hons.) Mathematics – DSC

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. Each part carries **7.5** marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve

$$\{y^2(x+1)+y\}dx + (2xy+1)dy = 0$$

P.T.O.

(b) Solve the initial value problem

$$x\frac{dy}{dx}\ y = y^2 \log x,\ \ y(1) = 2$$

(c) (i) Solve

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

(ii) Solve by reducing the order $y'' = (x + y')^2$

2. (a) Suppose that a mineral body formed in an ancient cataclysm originally contained the uranium isotope ^{238}U, (which has a half-life of 4.51×10^9 years) but no lead, the end product of the radioactive decay of ^{238}U. If today the ratio of ^{238}U atoms to lead atoms in the mineral body is 0.9, when did the cataclysm occur?

(b) Upon the birth of their first child, a couple deposited Rs. 10,000 in an account that pays 8% interest compounded continuously. The interest payment is allowed to accumulate. In how many years will the amount double? How much will the account contain on the child's 18^{th} birthday?

(c) A roast initially at 50°F, is placed in a 375°F oven at 5 pm. After 75 minutes, it is found that the temperature of the roast is 125°F. When will the roast be 150°F?

3. (a) Show that the solutions e^{x}, e^{-x}, e^{-2x} of the third order differential equation

$$\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$$

are linearly independent. Find the particular solution satisfying the given initial condition.

$$y(0)=1,\ y'(0)=2,\ y''(0)=0$$

(b) Solve the differential equation using the method of Variation of Parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x}$$

(c) Find the general solution of the differential equation using the method of Undetermined Coefficients.

$$\frac{d^2y}{dx^2} + 9y = 2\cos 3x$$

4. (a) Use the operator method to find the general solution of the following linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$

(b) Find the general solution of the differential equation. Assume $x > 0$.

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=72x^5$$

(c) A body with mass $m = \frac{1}{5}$ kg is attached to the end of a spring that is stretched 4m by a force of 20N. It is set in motion with initial position $x_0 = 1$m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of oscillation.

5. (a) Develop a model with two differential equations describing a predator-prey interaction.

(b) Define equilibrium solution of differential equation. A model of a three species interaction with two predators that compete for a single prey food-source is

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ, \frac{dY}{dt} = a_2XY - b_2Y, \frac{dZ}{dt} = a_3XZ - b_3Z$$

where a_i, b_i, c_i for $i = 1,2,3$ are all positive constants. Here X(t) is the prey density and Y(t), Z(t) are the two predator densities. Find all possible equilibrium populations.

(c) Suppose a population can be modeled using the differential equation

$$\frac{dX}{dt} = 0.2X - 0.001X^2$$

with an initial population size of $x_0 = 100$ and a time step of 1 month. Find the predicted population after 2 months.

6. (a) The Earth's atmospheric pressure p is often modelled by assuming that $\frac{dp}{dx}$ (the rate at which pressure p changes with altitude h above sea level) is proportional to p. Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 20 km is 50 millibars. Use an exponential decay model

$$\frac{dp}{dx} = kp$$

to describe the system, and then by solving the equation, find an expression for p in terms of h. Determine k and the constant of integration from the initial conditions. What is the atmospheric pressure at an altitude of 50 km?

(b) Discuss Phase Plane Analysis of predator-prey model.

(c) A large tank contains 100 litres of salt water. Initially s_0 kg of salt is dissolved. Salt water flows into the tank at the rate of 10 litres per minute, and the concentration $c_{in}(t)$ (kg of salt/litre) of this incoming water-salt mixture varies with time. We assume that the solution in the tank is thoroughly mixed and that the salt solution flows out at the same rate at which it flows in: that is, the volume of water-salt mixture in the tank remains constant. Find a differential equation for the amount of salt in the tank at any time t.